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Adaptive Control Designed via Deterministic Excitation*

ZHANG Jifeng
(Institute of Systems Science, Academia Sinica·Beijing, 100080, PRC)
(Department of Electrical Engineering, McGill University·Canada)
CHEN Hanfu
(Institute of Systems Science, Academia Sinica·Beijing, 100080, PRC)

Abstract: This paper considers the parameter estimation and adaptive stabilization problems for linear discrete-time systems with unknown parameters and bounded isturbances. The a-priori knowledge for designing adaptive controllers is only the order of the system. No assumption is required except controllability and observability of the system. The excitation signals are deterministic, and hence, no external stochastic excitation signal is applied.

Key words: adaptive control; deterministic excitation; stabilization; discrete-time

1 Introduction

Consider the linear single-input single-output discrete-time system

$$A(z)y_n = zB(z)u_n + w_n, \quad \forall\, n \geqslant 0, \tag{1.1}$$

where y_n, u_n and w_n are the system output, input and unknown disturbance, respectively, $A(z)$ and $B(z)$ are polynomials in backward shift operator z

$$A(z) = 1 + a_1 z + \cdots + a_p z^p, \quad p \geqslant 0, \quad a_p \neq 0, \tag{1.2}$$

$$B(z) = b_1 + \cdots + b_q z^{q-1}, \quad q \geqslant 1, \quad b_q \neq 0 \tag{1.3}$$

and

$$\theta = [-a_1 \quad \cdots \quad -a_p \quad b_1 \quad \cdots \quad b_q]^{\mathrm{T}} \tag{1.4}$$

is the unknown parameter of the system. The disturbance w_n is of arbitrary nature: deterministic or stochastic. Assume that $\{w_n\}$ satisfies the following long run average condition

$$\sup_{n\geqslant 0} \frac{1}{n+1}\sum_{j=0}^{n} w_j^2 < \infty, \tag{1.5}$$

or satisfies the more restrictive condition

$$\sup_{n\geqslant 0} |w_n| < \infty. \tag{1.6}$$

The problem of adaptive stabilization consists in designing control aiming at stabilizing the system with unknown parameters. For system (1.1) with $w_n \equiv 0$, the problem was discussed in [1~4] and others. When w_n is not identically equal to zero, the problem is usually solved under conditions more than coprimeness of $A(z)$ and $zB(z)$, which as well-known is sufficient for non-

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adaptive stabilization [5~8]. To the authors' knowledge, under the coprimeness condition only, the problem has first been solved in [9] for system (1. 1) with $\{w_n\}$ being a martingale difference sequence. As in many previous works summarized by Chen and Guo[10], the excitation signals used in [9] are stochastic processes, which, generally speaking, are more difficult to deal with than deterministic ones.

In this paper, under the assumption that $A(z)$ and $zB(z)$ are coprime, we give adaptive controls via deterministic excitation signal such that

$$\sup_{n\geq 0}\frac{1}{n+1}\sum_{j=1}^{n}(y_j^2+u_j^2)<\infty \tag{1.7}$$

for the case where (1. 5) holds and

$$\sup_{n\geq 0}(|y_n|+|u_n|)<\infty \tag{1.8}$$

for the case where (1. 6) is satisfied.

Through out the paper, for a polynomial $X(z)=\sum_{i=0}^{\mu}x_iz^i$, the norms $\|\cdot\|_1$ and $\|\cdot\|_2$ are defined as follows

$$\|X(z)\|_1=\sum_{i=0}^{\mu}|x_i| \quad\text{and}\quad \|X(z)\|_2=\Big(\sum_{i=0}^{\mu}|x_i|^2\Big)^{1/2}.$$

2 Estimation and Adaptive Control

We estimate the unknown parameter θ by the LS algorithm which recursively defines the estimate θ_n as follows:

$$\theta_{n+1}=\theta_n+\mu_nP_n\varphi_n(y_{n+1}^{\mathrm{T}}-\varphi_n^{\mathrm{T}}\theta_n), \tag{2.1}$$

$$P_{n+1}=P_n-\mu_nP_n\varphi_n\varphi_n^{\mathrm{T}}P_n,\quad \mu_n=(1+\varphi_n^{\mathrm{T}}P_n\varphi_n)^{-1}, \tag{2.2}$$

$$\varphi_n^{\mathrm{T}}=[y_n\ \cdots\ y_{n-p+1}\ u_n\ \cdots\ u_{n-q+1}] \tag{2.3}$$

with $P_0=I$ and arbitrary initial value

$$\theta_0^{\mathrm{T}}=[-a_{10}\ \cdots\ -a_{p0}\ b_{10}\ \cdots\ b_{q0}].$$

For any $n\geq 0$ write θ_n in the component form

$$\theta_n^{\mathrm{T}}=[-a_{1n}\ \cdots\ -a_{pn}\ b_{1n}\ \cdots\ b_{qn}]. \tag{2.4}$$

If $A(z)$ and $zB(z)$ are coprime, then there exist two polynomials

$$G(z)=1+\sum_{j=1}^{q-1}g_jz^j,\quad H(z)=\sum_{j=0}^{p-1}h_jz^j, \tag{2.5}$$

such that

$$A(z)G(z)-zB(z)H(z)=1. \tag{2.6}$$

Replacing a_i, b_j, g_k, h_s by their estimates a_{in}, b_{jn}, g_{kn} and h_{sn} respectively in (1. 2), (1. 3), (2. 5), $i=1,\cdots,p$, $j=1,\cdots,q$, $k=1,\cdots,q-1$, $s=0,\cdots,p-1$, we correspondingly denote $A(z)$, $B(z)$, $G(z)$ and $H(z)$ by $A_n(z)$, $B_n(z)$, $G_n(z)$ and $H_n(z)$, respectively, for example, $A_n(z)=1+a_{1n}z+\cdots+a_{pn}z^p$.

We need the following two lemmas proved in Chen and Zhang[9].

Lemma 1 If $A(z)$ and $zB(z)$ are coprime, then there is a constant $\varepsilon_\theta>0$ such that for any θ_n satisfying $\|\theta_n-\theta\|\leq\varepsilon_\theta$, the following Bezout equation

$$A_n(z)G_n(z) - zB_n(z)H_n(z) = 1, \tag{2.7}$$

has a unique solution $(G_n(z), H_n(z))$ satisfying

$$\deg(G_n(z)) \leqslant q-1, \quad \deg(H_n(z)) \leqslant p-1 \tag{2.8}$$

and

$$\|G_n(z)\|_i^i + \|H_n(z)\|_i^i \leqslant 1 + \|G(z)\|_i^i + \|H(z)\|_i^i, \tag{2.9}$$

for $i=1$ or 2.

Lemma 2 Let $\{w_n\}$ in (1.1) be any disturbance (deterministic or stochastic) satisfying (1.5). Then the LS estimate θ_n for θ has the following properties

$$\|\theta_n - \theta\|^2 \leqslant \frac{\|\theta - \theta_0\|^2 + 2nW}{\lambda_{\min}^{(n-1)}}, \quad \forall n \geqslant 0, \tag{2.10}$$

where $W \triangleq \sup\limits_{n\geqslant 0} \frac{1}{n+1}\sum\limits_{j=0}^{n} w_j^2 < \infty$ by condition (1.5) or (1.6), and $\lambda_{\min}^{(n)}$ denoles the minimum eigenvalue of $P_{n+1}^{-1} \triangleq I + \sum\limits_{i=0}^{n} \varphi_i\varphi_i^{\mathrm{T}}$.

From (2.6) it is clear that

$$\begin{aligned} y_n &= A(z)G(z)y_n - zB(z)H(z)y_n \\ &= G(z)[A(z)y_n - zB(z)u_n] + zB(z)[G(z)u_n - H(z)y_n] \\ &= G(z)w_n + zB(z)[G(z)u_n - H(z)y_n] \end{aligned} \tag{2.11}$$

and

$$u_n = H(z)w_n + A(z)[G(z)u_n - H(z)y_n]. \tag{2.12}$$

From this we see that in the case where θ is known and w_n is bounded in the sense (1.5) or (1.6), the system will be stabilized in the sense of (1.7) or (1.8) if u_n is defined from

$$G(z)u_n - H(z)y_n = 0. \tag{2.13}$$

The "certainty equivalence principle" suggests to us defining adaptive control from

$$G_n(z)u_n - H_n(z)y_n = 0. \tag{2.14}$$

However, in the present case the closeness of θ_n to θ is not guaranteed. Consequently, it is not clear if (2.7) is solvable or not. Even if $G_n(z)$ and $H_n(z)$ can be defined from (2.7) we still do not know whether or not they are close to $G(z)$ and $H(z)$ respectively. So it is important that θ_n somehow approximates θ. If this is the case, then adaptive control defined by (2.14) may hopefully stabilize the system. By lemma 2 we see that for first step of approximating θ we may apply an explosive excitation input, by which we mean such an input that yields $\lambda_{\min}^{(n)}/n \xrightarrow[n\to\infty]{} \infty$. However, the stabilization purpose (1.7) or (1.8) does not allow us to apply such an input for a period longer than finite. Thus we need to define stopping times σ_i at which we turn off the explosive excitation input and switch on the control defined by the certainty equivalence principle until τ_i at which the accuracy of the LS estimate θ_n becomes unsatisfactory and we have to apply the explosive excitation input again. After defining stopping times

$$0 \triangleq \tau_0 < \sigma_1 < \tau_1 < \sigma_2 < \tau_2 < \cdots,$$

it is most important to show that there is some integer i such that $\sigma_i < \infty$ and $\tau_i = \infty$, because otherwise the requirement (1. 7) or (1. 8) will never be met.

Let $\{\varepsilon_n\}$ be a real sequence with the following properties

$$0 < \varepsilon_n < 1, \quad \varepsilon_n \to 0, \quad \varepsilon_n^2 \alpha^n \geqslant 1, \tag{2.15}$$

where $\alpha > 1$ is chosen arbitrarily.

We now consider the case where (1. 5) holds.

Define stopping times as follows: $\tau_0 = 0$, and for any $i \geqslant 1$,

$$\begin{aligned} \sigma_i = \min\{n > \tau_{i-1}: & \sum_{j=0}^{n-1} \varphi_j \varphi_j^{\mathrm{T}} - n^2 \varepsilon_n^{-2} I > 0; \\ & (2.7) \text{ subject to } (2.8) \text{ is solvable}, \\ & \| G_n(z) \|_2^2 + \| H_n(z) \|_2^2 \leqslant \frac{1}{\gamma \varepsilon_n}; \quad \text{and} \\ & \sum_{j=0}^{n-1} (y_j - \varphi_{j-1}^{\mathrm{T}} \theta_n)^2 \leqslant \varepsilon_n^2 s_n(\alpha^{2n}) \}, \end{aligned} \tag{2.16}$$

$$\tau_i = \min\{n > \sigma_i: \sum_{j=0}^{n-1} (y_j - \varphi_{j-1}^{\mathrm{T}} \theta_{\sigma_i})^2 > \varepsilon_{\sigma_i}^2 s_n(\alpha^{2\sigma_i})\}, \tag{2.17}$$

where $\gamma = \max\{p, q\}$ and $s_n(x)$ is given by $s_0(x) = 1$,

$$s_n(x) = n\max\{x, \frac{1}{k}\sum_{j=0}^{k-1}(y_j^2 + u_j^2), \quad k = 1, \cdots, n\}, \quad \forall\, n \geqslant 1. \tag{2.18}$$

Finally, adaptive control u_n at time n is given by

$$u_n = \begin{cases} \alpha^n, & \text{if } n \in [\tau_i, \sigma_{i+1}) \text{ and } n = \tau_i + 2k(p+q) + p + q \text{ for some } i \geqslant 0 \text{ and } k \geqslant 0; \\ 0, & \text{if } n \in [\tau_i, \sigma_{i+1}) \text{ for some } i \geqslant 0, \text{ but} \\ & \quad n \neq \tau_i + 2k(p+q) + p + q \text{ for all } k \geqslant 0; \\ H_{\sigma_i}(z) y_n - (G_{\sigma_i}(z) - 1) u_n, & \text{if } n \in [\sigma_i, \tau_i) \text{ for some } i \geqslant 1. \end{cases} \tag{2.19}$$

In the following lemma we introduce a deterministic excitation signal which is much simpler to be proved explosive in comparison with the stochastic one used in [9] and [11].

Lemma 3 If $A(z)$ and $zB(z)$ are coprime, (1. 5) holds and

$$u_n = \begin{cases} \alpha^n, & \text{if } n = 2k(p+q) + p + q \text{ for } k = 0, 1, \cdots, \\ 0, & \text{otherwise}, \end{cases} \tag{2.20}$$

where $\alpha > 1$ can be arbitrarily chosen, then for any $n \geqslant 2(p+q)$,

$$\lambda_{\min}^{(n)} \geqslant \frac{\varepsilon}{2C} \alpha^{2n - 6(p+q)} - pC^{-1} W_n, \tag{2.21}$$

with $C = (p+1)(1 + \sum_{j=1}^{p} a_j^2)$, ε and W defined in (2. 24) below.

Proof Set $\Phi_n = A(z)\varphi_n$ and $D = [D_1, D_2]^{\mathrm{T}}$, where

$$D_1^{\mathrm{T}} = \overbrace{\begin{bmatrix} 0 & b_1 & \cdots & \cdots & \cdots & b_q & 0 & \cdots & 0 \\ 0 & 0 & \ddots & & & & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & & & & \ddots & 0 \\ 0 & \cdots & 0 & 0 & b_1 & \cdots & \cdots & \cdots & b_q \end{bmatrix}}^{p+q} \left.\vphantom{\begin{bmatrix} 0\\0\\0\\0 \end{bmatrix}}\right\} p$$

and

$$D_2^{\mathrm{T}} = \left.\overbrace{\begin{bmatrix} 1 & a_1 & \cdots & \cdots & \cdots & \cdots & a_p & 0 & \cdots & 0 \\ 0 & 1 & \ddots & & & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & & & & & \ddots & 0 \\ 0 & \cdots & 0 & 0 & a_1 & \cdots & \cdots & \cdots & \cdots & a_p \end{bmatrix}}^{p+q}\right\} q.$$

From (1.1) it is easy to see that

$$\Phi_n = DU_n + W_n, \tag{2.22}$$

where

$$U_n = [u_n \quad \cdots \quad u_{n-(p+q)+1}]^{\mathrm{T}}, \ W_n = [w_n \cdots \ w_{n-p+1} \ \underbrace{0 \cdots 0}_{q}]^{\mathrm{T}}. \tag{2.23}$$

Let k be the largest integer such that $2(k+1)\ (p+q) \leqslant n$, and set $n_k = 2k(p+q)$. Then it is not difficult to see that for any $\eta \in \mathrm{R}^{p+q}$ with $\|\eta\| = 1$,

$$\sum_{i=n_k+p+q}^{n_k+2(p+q)-1} \|\eta^{\mathrm{T}}\Phi_i\|^2 \geqslant \frac{1}{2} \sum_{i=n_k+p+q}^{n_k+2(p+q)-1} \|\eta^{\mathrm{T}}DU_i\|^2 - \sum_{i=n_k+p+q}^{n_k+2(p+q)-1} \|\eta^{\mathrm{T}}W_i\|^2,$$

which together with the fact $\varepsilon \triangleq \lambda_{\min}(DD^{\mathrm{T}}) = \lambda_{\min}(M^{\mathrm{T}}M) > 0$ implies that

$$\lambda_{\min}\Big(\sum_{i=n_k+p+q}^{n_k+2(p+q)-1} \Phi_i\Phi_i^{\mathrm{T}}\Big) \geqslant \frac{1}{2}\lambda_{\min}\Big(\sum_{i=n_k+p+q}^{n_k+2(p+q)-1} DU_iU_i^{\mathrm{T}}D^{\mathrm{T}}\Big) - \lambda_{\max}\Big(\sum_{i=n_k+p+q}^{n_k+2(p+q)-1} W_iW_i^{\mathrm{T}}\Big)$$

$$\geqslant \frac{1}{2}\lambda_{\min}(DD^{\mathrm{T}})\lambda_{\min}\Big(\sum_{i=n_k+p+q}^{n_k+2(p+q)-1} U_iU_i^{\mathrm{T}}\Big) - p\sum_{i=n_k+q}^{n_k+2(p+q)-1} \|w_i\|^2$$

$$\geqslant \frac{1}{2}\varepsilon\lambda_{\min}\Big(\sum_{i=n_k+p+q}^{n_k+2(p+q)-1} U_iU_i^{\mathrm{T}}\Big) - p(n_k + 2p + 2q)W, \tag{2.24}$$

where $W \triangleq \sup\limits_{n\geqslant 0} \dfrac{1}{n+1}\sum\limits_{j=0}^{n} w_j^2 < \infty$ by condition (1.5) or (1.6).

On the other hand, we have

$$\lambda_{\min}\Big(\sum_{i=n_k+p+q}^{n_k+2(p+q)-1} \Phi_i\Phi_i^{\mathrm{T}}\Big) = \inf_{\|x\|=1} \sum_{i=n_k+p+q}^{n_k+2(p+q)-1} (x^{\mathrm{T}}\Phi_i)^2$$

$$\leqslant \lambda_{\min}\Big(\sum_{i=n_k+q}^{n_k+2(p+q)-1} \varphi_i\varphi_i^{\mathrm{T}}\Big)\Big[(p+1)\Big(1 + \sum_{j=1}^{p} a_j^2\Big)\Big],$$

which together with (2.24) yields

$$\lambda_{\min}\Big(\sum_{i=n_k+q}^{n_k+2(p+q)-1} \varphi_i\varphi_i^{\mathrm{T}}\Big) \geqslant \frac{\varepsilon}{2C}\lambda_{\min}\Big(\sum_{i=n_k+p+q}^{n_k+2(p+q)-1} U_iU_i^{\mathrm{T}}\Big) - pC^{-1}Wn, \tag{2.25}$$

where

$$C = (p+1)\Big(1 + \sum_{j=1}^{p} a_j^2\Big).$$

From (2.20) it is easy to get that

$$\sum_{i=n_k+p+q}^{n_k+2(p+q)-1} U_iU_i^{\mathrm{T}} = \alpha^{2(n_k+p+q)} I_{(p+q)\times(p+q)} \geqslant \alpha^{2n-6(p+q)} I_{(p+q)\times(p+q)}.$$

From this and (2.25) we obtain (2.21).

3 Main Results

Theorem 1 If $A(z)$ and $zB(z)$ are coprime, and disturbance $\{w_n\}$ is bounded in the sense (1.5), then the adaptive control (2.19) stabilizes the closed-loop system in the following sense

$$\sup_{n\geqslant 0}\frac{1}{n+1}\sum_{j=0}^{n}(y_j^2+u_j^2)<\infty \tag{3.1}$$

for arbitrary initial values $y_i,\ i=0,-1,\cdots,-p,\ u_j,\ j=0,-1,\cdots,-q$.

Proof The first step is to show that there exists an integer $i\geqslant 1$ such that $\sigma_i<\infty$ and $\tau_i=\infty$.

We now prove that it is impossible that $\tau_i<\infty$ and $\sigma_{i+1}=\infty$. In fact, if there were an $i\geqslant 0$ such that $\tau_i<\infty$ and $\sigma_{i+1}=\infty$, then by (2.19) we get

$$u_n=\begin{cases}\alpha^n, & \text{if } n=\tau_i+2k(p+q)+p+q \text{ for some } k\geqslant 0;\\ 0, & \text{if } n\geqslant\tau_i, \quad \text{but } n\neq\tau_i+2k(p+q)+p+q \text{ for all } k\geqslant 0.\end{cases} \tag{3.2}$$

Hence, by Lemmas 2 and 3 we would have that for any $n\geqslant\tau_i+2(p+q)$,

$$\|\tilde{\theta}_n\|^2\leqslant\frac{\|\tilde{\theta}_0\|^2+2Wn}{\lambda_{\min}^{(n-1)}} \text{ and } \lambda_{\min}^{(n)}\geqslant\frac{\varepsilon}{2C}\alpha^{2n-6(p+q)}-pC^{-1}Wn, \tag{3.3}$$

where
$$\tilde{\theta}_n=\theta-\theta_n.$$

From this, Lemma 1 and (2.15) we see that all requirements except the last inequality listed in (2.16) are met for all $n\geqslant N_0$ starting from some integer $N_0\geqslant\tau_i+2(p+q)$.

Set $C_0=\sum_{j=-\gamma}^{0}(y_j^2+u_j^2)$. Then by (1.1), (2.18), (3.3) and (1.5) we obtain that for any $n\geqslant N_0$,

$$\begin{aligned}\sum_{j=0}^{n-1}(y_j-\varphi_{j-1}^{\mathrm{T}}\theta_n)^2&\leqslant 2\sum_{j=0}^{n-1}(\varphi_{j-1}^{\mathrm{T}}\tilde{\theta}_n)^2+2\sum_{j=0}^{n-1}w_j^2\\ &\leqslant 2\gamma[s_n(\alpha^{2n})+C_0]\|\theta_n\|^2+2W_n\\ &\leqslant s_n(\alpha^{2n})\left(1+\frac{C_0}{\alpha^{2n}}\right)\left(\frac{2\gamma[\|\tilde{\theta}_0\|^2+2Wn]}{\varepsilon(2C)^{-1}\alpha^{2(n-1)-6(p+q)}-pC^{-1}Wn}+\frac{2W}{\alpha^{2n}}\right),\end{aligned} \tag{3.4}$$

which together (2.15) implies that there exists an integer $N_1\geqslant N_0$ such that for any $n\geqslant N_1$

$$\sum_{j=0}^{n-1}(y_j-\Phi_{j-1}^{\mathrm{T}}\theta_n)^2\leqslant\varepsilon_n^2 s_n(\alpha^{2n}).$$

Therefore, we have $\sigma_{i+1}\leqslant N_1$. This contradicts $\sigma_{i+1}=\infty$.

We now prove that $\tau_i=\infty$ for some i.

By Lemma 2 we see that

$$\|\tilde{\theta}_{\sigma_i}\|^2\leqslant\frac{\|\tilde{\theta}_0\|^2+2W\sigma_i}{\lambda_{\min}^{(\sigma_i-1)}},$$

which incorporating the definition of σ_i implies that

$$\|\tilde{\theta}_{\sigma_i}\|^2\leqslant\varepsilon_{\sigma_i}^2\frac{\|\tilde{\theta}_0\|^2+2W\sigma_i}{\sigma_i^2}. \tag{3.5}$$

Similar to (3.4), by (3.5), (2.15), (2.18) we obtain that

$$\sum_{j=0}^{n-1}(y_j-\varphi_{j-1}^{\mathrm{T}}\theta_{\sigma_i})^2\leqslant 2\sum_{j=0}^{n-1}(\varphi_{j-1}^{\mathrm{T}}\tilde{\theta}_{\sigma_i})^2+2\sum_{j=0}^{n-1}w_j^2$$

$$\leqslant \varepsilon_{\sigma_i}^2 s_n(\alpha^{2\sigma_i}) \left[\left(1 + \frac{C_0}{\alpha^{2\sigma_i}}\right) + \frac{\| \tilde{\theta}_0 \|^2 + 2W\sigma_i}{\sigma_i^2} + \frac{2W}{\alpha^{\sigma_i}} \right], \tag{3.6}$$

which together with (3.5) and (2.15) implies that for some large enough $i \geqslant 1$ and any $n \geqslant \sigma_i$, one has

$$\sum_{j=0}^{n-1} (y_j - \varphi_{j-1}^{\mathrm{T}} \theta_{\sigma_i})^2 \leqslant \varepsilon_{\sigma_i}^2 s_n(\alpha^{2\sigma_i}).$$

Therefore, there must be an i for which $\tau_i = \infty$.

The second step is to prove (3.1) by use of the fact that for some i, $\sigma_i < \infty$ and $\tau_i = \infty$.

By (2.7) we have

$$y_n = G_{\sigma_i}(z)[A_{\sigma_i}(z)y_n - zB_{\sigma_i}(z)u_n] + zB_{\sigma_i}(z)[G_{\sigma_i}(z)u_n - H_{\sigma_i}(z)y_n],$$

$$u_n = H_{\sigma_i}(z)[A_{\sigma_i}(z)y_n - zB_{\sigma_i}(z)u_n] + A_{\sigma_i}(z)[G_{\sigma_i}(z)u_n - H_{\sigma_i}(z)y_n].$$

Hence, from (2.19) we get, for any $n \geqslant n_0 \triangleq \sigma_i + \max(p, q)$,

$$y_n = G_{\sigma_i}(z)[A_{\sigma_i}(z)y_n - zB_{\sigma_i}(z)u_n], \tag{3.7}$$

$$u_n = H_{\sigma_i}(z)[A_{\sigma_i}(z)y_n - zB_{\sigma_i}(z)u_n]. \tag{3.8}$$

From (3.7) and (3.8) it follows that for any $n \geqslant n_0$,

$$\frac{1}{n}\sum_{j=0}^{n-1}(y_j^2 + u_j^2) = \frac{1}{n}\sum_{j=n_0}^{n-1}(y_j^2 + u_j^2) + \frac{1}{n}\sum_{j=0}^{n_0-1}(y_j^2 + u_j^2)$$

$$\leqslant \frac{\gamma}{n}(\| G_{\sigma_i}(z) \|_2^2 + \| H_{\sigma_i}(z) \|_2^2) \sum_{j=0}^{n-1}(y_j - \varphi_{j-1}^{\mathrm{T}}\theta_{\sigma_i})^2 + \frac{1}{n}\sum_{j=0}^{n_0-1}(y_j^2 + u_j^2)$$

$$\leqslant \frac{1}{\varepsilon_{\sigma_i}} \frac{1}{n} \sum_{j=0}^{n-1}(y_j - \varphi_{j-1}^{\mathrm{T}}\theta_{\sigma_i})^2 + c_1 \leqslant \varepsilon_{\sigma_i} \frac{s_n(\alpha^{2\sigma_i})}{n} + c_1, \tag{3.9}$$

where
$$c_1 = \sum_{j=0}^{n_0-1}(y_j^2 + u_j^2).$$

Noticing that $\frac{s_n(\alpha^{2\sigma_i})}{n}$ is nondecreasing from (3.9) we get for any $n \geqslant n_0$ and any $l \in [n_0, n]$,

$$\frac{1}{l}\sum_{j=0}^{l-1}(y_j^2 + u_j^2) \leqslant \varepsilon_{\sigma_i}\frac{s_l(\alpha^{2\sigma_i})}{l} + c_1 \leqslant \varepsilon_{\sigma_i}\frac{s_n(\alpha^{2\sigma_i})}{n} + c_1,$$

which together with (2.18) yields

$$\frac{s_n(\alpha^{2\sigma_i})}{n} \leqslant \max\left\{ \alpha^{2\sigma_i}; \frac{1}{l}\sum_{j=0}^{l-1}(y_j^2 + u_j^2), \quad l = 1, \cdots, n_0 - 1; \ \varepsilon_{\sigma_i}\frac{s_n(\alpha^{2\sigma_i})}{n} + c_1 \right\}. \tag{3.10}$$

Set

$$c_2 = \alpha^{2\sigma_i} + c_1 + \max\left\{ \frac{1}{l}\sum_{j=0}^{l-1}(y_j^2 + u_j^2), \quad l = 1, \cdots, n_0 - 1 \right\}.$$

Then (3.10) implies that for any $n \geqslant 1$,

$$\frac{s_n(\alpha^{2\sigma_i})}{n} \leqslant \varepsilon_{\sigma_i}\frac{s_n(\alpha^{2\sigma_i})}{n} + c_2,$$

which means

$$\frac{s_n(\alpha^{2\sigma_i})}{n} \leqslant (1 - \varepsilon_{\sigma_i})^{-1} c_2,$$

i. e. $s_n(\alpha^{2\sigma_i})n$ is bounded, and hence, (3. 1) is true. Q. E. D.

We now consider the case where (1. 6) holds.

Define stopping times as follows: $\tau_0=0$, and for any $i\geqslant 1$,

$$\sigma_i = \min\{n > \tau_{i-1}: \sum_{j=0}^{n-1}\varphi_j\varphi_j^{\mathrm{T}} - n^2\varepsilon_n^{-1}I > 0;$$

(2. 7) subject to (2. 8) is solvable,

$$\| G_n(z) \|_1 + \| H_n(z) \|_1 \leqslant \frac{1}{2\gamma\varepsilon_n}$$

and
$$|y_n - \varphi_{n-1}^{\mathrm{T}}\theta_n| \leqslant \varepsilon_n s_n'(\alpha^{2n})\}, \tag{3. 11}$$

$$\tau_i = \min\{n > \sigma_i: |y_n - \varphi_{n-1}^{\mathrm{T}}\theta_{\sigma_i}| > \varepsilon_{\sigma_i} s_n'(\alpha^{2n})\}, \tag{3. 12}$$

where $\gamma=\max\{p,q\}$ and $s_n'(x)$ is given by $s_0'(x)=1$,

$$s_n'(x) = \max\{x,\ |y_j|,\ |u_j|,\ j = n-\gamma,\ \cdots,\ n-1\},\quad \forall\ n \geqslant 1. \tag{3. 13}$$

Theorem 2 If $A(z)$ and $zB(z)$ are coprime, and disturbance $\{w_n\}$ is bounded in the sense (1. 6), then the adaptive control (2. 19) with σ_i, τ_i given by (3. 11)~(3. 13) stabilizes the closed-loop system in the following sense

$$\sup_{n\geqslant 0}(|y_n| + |u_n|) < \infty \tag{3. 14}$$

for arbitrary initial values y_i, $i=0,-1,\cdots,-p$, u_j, $j=0,-1,\cdots,-q$.

Proof Similar to the argument of Theorem 1 we can show that there is an integer i such that $\sigma_i<\infty$ and $\tau_i=\infty$. Therefore, for any $n_1\triangleq\sigma_i+\gamma$, (3. 7) and (3. 8) hold, and for any $n\geqslant\sigma_i$,

$$|y_n - \varphi_{n-1}^{\mathrm{T}}\theta_{\sigma_i}| \leqslant \varepsilon_{\sigma_i} s_n'(\alpha^{2\sigma_i}). \tag{3. 15}$$

From (3. 7) and (3. 15) we see that for any $n\geqslant n_1$,

$$\begin{aligned}|y_n| &= |G_{\sigma_i}(z)(y_n - \varphi_{n-1}^{\mathrm{T}}\theta_{\sigma_i})| \\ &\leqslant \| G_{\sigma_i}(z) \|_1 \max_{0\leqslant j\leqslant p-1} |y_{n-j} - \varphi_{n-1-j}^{\mathrm{T}}\theta_{\sigma_i})| \\ &\leqslant \varepsilon_{\sigma_i} \| G_{\sigma_i}(z) \|_1 \max_{0\leqslant j\leqslant p-1} s_{n-j}'(\alpha^{2\sigma_i}).\end{aligned} \tag{3. 16}$$

Similarly, from (3. 8) and (3. 15) we get

$$|u_n| \leqslant \varepsilon_{\sigma_i} \| H_{\sigma_i}(z) \|_1 \max_{0\leqslant j\leqslant q-1} s_{n-j}'(\alpha^{2\sigma_i}),$$

which together with (3. 16) and

$$\| G_{\sigma_i}(z) \|_1 + \| H_{\sigma_i}(z) \|_1 \leqslant \frac{1}{2\gamma\varepsilon_{\sigma_i}},$$

yields

$$\max\{|y_n|,\ |u_n|\} \leqslant \frac{1}{2\gamma} \max_{0\leqslant j\leqslant \gamma-1} s_{n-j}'(\alpha^{2\sigma_i}).$$

From this and (3. 13) it is not difficult to see that

$$s_{n+2\gamma}'(\alpha^{2\sigma_i}) \leqslant \alpha^{2\sigma_i} + \frac{1}{2\gamma}\sum_{j=1}^{2\gamma-1} s_{n+2\gamma-j}'(\alpha^{2\sigma_i}),$$

which together with Lemma 3 in [4] implies that

$$\sup_{n\geq 0} s'_n(\alpha^{2\sigma_i}) \leqslant c\alpha^{2\sigma_i} < \infty,$$

where c is a constant and depends on γ only. Q. E. D.

Remark Both Theorems 1 and 2 conclude that there is an integer $i\geqslant 1$ such that $\sigma_i<\infty$ and $\tau_i=\infty$ and for $n>\sigma_i$ the adaptive control is defined from

$$H_{\sigma_i}(z)y_n - G_{\sigma_i}(z)u_n = 0.$$

This together with (1. 1) implies that after a finite number of steps the closed-loop system eventually becomes

$$F(z)y_n = G_{\sigma_i}(z)w_n \quad \text{with} \quad F(z) = A(z)G_{\sigma_i}(z) - zB(z)H_{\sigma_i}(z).$$

It is clear that σ_i, and hence, $F(z)$ depends on $\{w_n\}$.

4 Conclusion Remarks

For a single-input single-output discrete-time system with unknown parameters and bounded disturbances; an indirect adaptive stabilization controller is presented. The construction of the controller is characterized by a deterministic excitation signal sequence and an appropriate time splitting. The a-priori knowledge for designing adaptive controllers is only the order of the system. No matter what the feature of $w(t)$ is, deterministic or stochastic, the adaptive controller stabilizes the closed-loop system. Hence, it is possible to deal with adaptive control problems by use of a unified algorithm, for bath deterministic and stochastic systems.

Reference

[1] Praly, L.. Towards a Globally Stable Direct Adaptive Control Scheme for Not Necessarily Minimum Phase Systems. IEEE Trans. Automat. Contr., 1984, AC-29(10):946—949

[2] Elliott, H., Cristi, R. and Das, M.. Global Stability of Adaptive Pole Placement Algorithms. IEEE Trans. Automat. Contr., 1985, AC-30(3):348—356

[3] Kreisselmeier, G.. An Indirect Adaptive Controller with a Self-Excitation Capability. IEEE Trans. Automat. Contr., 1989, AC-34(5):524—528

[4] Chen, H. F. and Zhang, J. F. Adaptive Regulation for Deterministic Systems. Acta Mathematicae Applicatae Sinica, 1991, 7(4):332—343

[5] Egardt, B. and Samson, C.. Stable Adaptive Control of Non-Minimum Phase Systems. Systems & Control Letters, 1982, 2(3):137—144

[6] Giri, F., M'Saad, M, Dugard, L. and Dion, J. M.. Robust Pole Placement Indirect Adaptive Control. Int. J. of Adaptive Control and Signal Processing, 1988, 2(1):33—47

[7] Giri, F., M'Saad, M, Dugard, L. and Dion, J. M.. A Cautious Approach to Robust Adaptive Regulation. Int. J. of Adaptive Control and Signal Processing, 1988, 2(4):273—290

[8] Lozano-Leal, R.. Robust Adaptive Regulation Without Persistent Excitation. IEEE Trans., Automat. Contr., 1989, AC-34(12):1260—1267

[9] Chen, H. F. and Zhang, J. F.. Adaptive Stabilization of Unstable and Nonminimum-Phase Stochastic Systems. Systems and Control Letters, 1993, 20(1):27—38

[10] Chen, H. F. and Guo, L.. Identification and Stochastic Adaptive Control. Birkhäuser, Boston, 1991

[11] Zhang, J. F. and Chen H. F.. Adaptive Stabilization under the Weakest Condition. Submitted for Publication, 1991

用确定性激励设计的适应控制

张纪峰

（中国科学院系统科学研究所，加拿大 McGill 大学电气工程系）

陈翰馥

（中国科学院系统科学研究所·北京，100080）

摘要：本文解决参数未知、干扰有界的线性离散时间系统的参数估计和适应镇定问题，设计控制器时事先只要求系统的阶已知，系统能控、能观测，设计时所用的外部激励不用随机信号而用确定性信号.

关键词：适应控制；确定性激励；镇定；离散时间系统

本文作者简介

张纪峰 1963 年生. 1985 年毕业于山东大学数学系，分别于 1988 年和 1991 年在中国科学院系统科学研究所获硕士及博士学位. 1991 年至 1992 年在加拿大 McGill 大学做博士后. 研究兴趣为随机系统及奇异系统，在国内外著名刊物发表论文 20 余篇. 曾两次荣获中国科学院院长奖学金优秀奖.

陈翰馥 1937 年生. 1961 年毕业于前苏联列宁格勒大学数学系，然后工作于中国科学院数学所，系统科学研究所. 曾在随机系统，过程统计方面发表论文一百余篇，近半数在国外刊物上发表，发表专著 5 本，其中两本在美国出版. 现任 IFAC 理论委员会副主席，中国数学学会及中国自动化学会常务理事. 研究领域为随机系统的辨识，控制，适应控制，递推估计及随机逼近等.